A Framework for Evaluating Physical-Layer Network Coding Gains in Multi-hop Wireless Networks

Raphaël Naves[†] [*], Hicham Khalifé[†], Gentian Jakllari[*], Vania Conan[†], and André-Luc Beylot[*]

[†]Thales Communications & Security, France - Email: name.surname@thalesgroup.com
[*]IRIT-INPT/ENSEEIHT, University of Toulouse, France - Email:name.surname@enseeiht.fr

***Abstract*—We investigate the potential gains of Physical-Layer Network Coding (PLNC) in multi-hop wireless networks. Physical-Layer Network Coding was first introduced as a solution to increase the throughput of a two-way relay channel communication. Unlike most wireless communications techniques which try to avoid collisions, PLNC allows two simultaneous transmissions to a common receiver. Such transmitted messages are summed at signal level and then decoded at packet level. In basic topologies, Physical-Layer Network Coding has been shown to significantly enhance the throughput performance compared to classical communications. However, the impact of PLNC in large multi-hop networks remains an open question. We therefore exploit Linear Programming to evaluate the impact of this paradigm in large realistic radio deployments. Our numerical results show that PLNC can increase the throughput in large multi-hop topologies by 30%. Such gains set theoretical benchmarks for designing new access methods and routing protocols to efficiently exploit the Physical-Layer Network Coding concept.**

I. Introduction

The constant increase in the number of devices as well as the exponential growth in the exchanged data volumes render the future networks highly dense in terms of nodes and conveyed traffic. In a wireless environment, this density challenges today's wireless communication techniques and requires new interference mitigation strategies.

Physical-Layer Network Coding (PLNC) [1], can reduce the interference by allowing multiple concurrent transmissions simultaneously. It was originally proposed in [2] as a solution to increase the throughput of a two-way relay channel communication (TWRC). In this simple scenario, two nodes A and B send a packet mutually through the same relay R (Fig. 1). Unlike most wireless communications techniques which try to avoid collisions, PLNC allows simultaneous transmissions of two messages to the shared relay. A and B can transmit in the same slot their packet (p_1 and p_2 respectively) to R. By decoding the two signals, naturally mixed at the physical level, R retrieves at the bit level a packet p_3 which is the result of a XOR operation between p_1 and p_2. In the second slot, R sends p_3 to the two end users, exploiting the broadcast nature of wireless medium. A and B can extract their intended message removing their contribution to the received packet thanks to a XOR operation with the message sent in the first slot. In this simple TWRC scenario, the Physical-Layer Network Coding theoretical throughput gains reach 100% and 50% compared to classical transmissions and traditional well-known Network Coding [3], respectively. This gain comes at the price of strict synchronization to guarantee the reception of perfectly aligned signals at the relay.

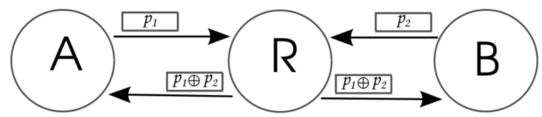


Fig. 1. The Two-Way Relay Model (TWRC) - 2 steps (time-slots) illustration

Some research initiatives have tried to investigate the performance of Physical-Layer Network Coding in multi-hop networks, however, the focus was on small topologies, using basic PLNC modes [4]. The contribution of our paper is twofold. We first propose a framework for precisely evaluating Physical-Layer Network Coding performance in large multi-hop networks. In our approach, we derive the achievable throughput for any given realistic topology with optimal scheduling using PLNC with TWRC model then compare to state of the art scheduling solutions. To do so, we define a new wireless transmission model that we formalize as a linear program. Second, driven by the observation that in many real multi-hop topologies with realistic traffic patterns the TWRC does not apply, we further propose and model, in our linear programs, new constraints for enabling Physical-Layer Network Coding beyond the classical TWRC model, encompassing the Butterfly and intra-flow PLNC (i.e encoding unidirectional flows). To our knowledge, no previous work has investigated the use of Physical-Layer Network Coding nor quantified the gains it can produce in realistic wireless deployments.

Our results show that Physical-Layer Network Coding can offer significant throughput gains in large topologies. Interestingly, these gains are even more significant when the number of flows increases. Besides, intra-flow PLNC allows to reduce interference in topologies where many unidirectional flows are competing. Such situations are typical when many relays upload traffic to a common sink. These theoretical results set the baselines for the achievable throughput using PLNC in large multi-hop network with optimal scheduling, highlighting its ability to operate in interference prone environments. Our work shall guide protocol designers when conceiving and evaluating MAC and routing solutions for PLNC.

The remainder of the paper is structured as follows: In Section II, we give an overview on previous work on throughput computation and Physical-Layer Network Coding. We then detail in Section III our system model and considered assumptions that are used in Section IV to define the linear programs constituting our framework. Section V shows the throughput gains achieved with the TWRC model whereas an extension of this model to more advanced Physical-Layer Network Coding schemes is highlighted in Section VI. Section VII concludes this paper and describes future research perspectives.

II. Related work

A. Scheduling and throughput calculation

During the last decade, scheduling concurrent transmissions in wireless networks have attracted a lot of attention from the research community. Indeed, a smart medium sharing between nodes is essential to ensure an efficient bandwidth utilization. But, this problem is known to be NP-complete with traditional models of transmissions ([5]). However, Jain *et al.* work in [6] is considered as an important breakthrough in this area. Indeed, the paper proposes a framework to precisely evaluate the end-to-end throughput of any concurrent unicast transmissions in any topology, using implicitly an optimal scheduling. Even if the solution does not define the scheduling to adopt, the work constitutes a strong basis to determine the theoretical achievable performance of any wireless networks. In their approach, the authors exploit the well-known protocol model (based only on distance) and physical model (based on distance and received signal strength) as interference models which ensure that each destination of a unicast transmission receives distinctly its intended packet. They thus model classical access techniques whose role is to avoid collision of packets. Integrating to an optimization problem the set of transmissions that can be scheduled in the same slot without interfering, an achievable lower bound on the throughput can then be computed. When all these different sets of links are found, this lower bound converges to the upper bound therefore defining the maximum achievable throughput. Clearly, the main challenge is to account for all these possible set of links what constitutes an NP-hard problem. However, authors claim that only few minutes are required to compute the achievable maximum throughput of any reasonable size networks (more than 20 nodes).

More recently, authors in [7] extended the previous optimization problem with the capability to illustrate the classical (packet based) network coding gains. To this aim, authors authorize broadcast communication schemes. The protocol model is also used to ensure a sufficient Signal Noise Ratio (SNR) at all receivers of broadcast communications. Two ways of using Network Coding are considered: with and without opportunistic listening. Interestingly, authors highlight that more than 30% gain in throughput can be achieved in certain conditions compared to simple unicast transmissions. These results confirm the intuitive idea that Network Coding, which was originally introduced for small topologies, offers promising performance in large ad-hoc networks. More importantly, the described framework offers a tool to precisely compute the maximum throughput per unicast flow, in any considered topology in the presence of network coding.

B. Physical-Layer Network Coding in large multi-hop networks

Although it has been proved that Physical-Layer Network Coding does not significantly impact the complexity of the scheduling problem [8], the gains it offers especially in large multi-hop and realistic topologies remains often unknown. Indeed, if the performance of PLNC no longer has to be proved in small topologies [4], the efficiency of this technique in large multi-hop networks remains an open question. To the best of our knowledge, no study has been conducted to accurately evaluate and quantify potential gains of PLNC in realistic ad-hoc networks. It has been showed in [9] and [10] that PLNC does not change the scaling law derived by Gupta and Kumar in [11]. More precisely, with an infinity of flows in the network, the per node throughput is improved with a fixed factor but converges also asymptotically to zero with PLNC. Undoubtedly, this result is interesting however, it does not characterize the theoretical benefits of PLNC in realistic radio deployments. Clearly, this information is crucial to understand the topologies where PLNC needs can applied and where it cannot.

More importantly, PLNC techniques have not been generalized to realistic multi-hop networks. Most of the studies focus on the TWRC scheme previously described. This observation can be made on both theoretical papers and implementation solutions ([12], [13]). The impressive gains that PLNC allows in this configuration encouraged researchers to integrate this two-hop scenario in large ad-hoc networks. In [14] for instance, an access method is especially designed to detect favorable conditions to set up a PLNC transmission when three nodes are selected in a way to form a TWRC scheme. Even if a global throughput increase can be observed with some particular traffic patterns, one can wonder whether the Physical-Layer Network Coding is exploited to its maximum potential. Indeed, the TWRC scenario fits perfectly to two-hops networks with bi-directional traffic, but is it still the case for large ad-hoc networks with concurrent multi-hop unicast flows? As highlighted in [2], other more ambitious models of PLNC models are possible. In fact, very recently, the work proposed in [15] and [16] integrates new PLNC schemes. In particular, authors combine PLNC and the opportunistic listening technique that wireless medium allows, to improve the performance of the network. What is more, the obtained results in the realistic deployments of [15], confirm the intuitive idea that different PLNC strategies can be more efficient in certain conditions than the basic TWRC model. Our paper addresses the following question: Are the 50% gain offered by PLNC claimed in the paper still achievable with other sizes of networks, other topologies, and other traffic models.

III. Modeling assumptions

We consider a classical multi-hop wireless topology where unicast packets are exchanged between a source and a destination. All nodes are assumed to be identical serving as source, relay, or destination of exchanged traffic. Quite logically, if the source and the destination are not direct neighbors (i.e. the received signal at the destination does not allow frame reception), the frame is routed through other nodes using assumed existing classical, often multi-path, routing solutions. We also assume a TDMA-based access control meaning that nodes are allowed to transmit in particular time slots with a synchronized shared clocking.

A. PLNC interference model

In the two works dealing with throughput calculation [6] and [7], the protocol model or the physical model are both considered for interference modelling. As commonly admitted in most of wireless communications strategies, they illustrate the idea that a transmission between two nodes cannot be received without error if nodes close to the destination are also transmitting. Obviously this assumption is no longer appropriate with PLNC transmissions. Clearly, the main concept of Physical-Layer Network Coding is to allow two simultaneous transmissions to a same receiver, as illustrated in the TWRC scheme (Fig. 1). It becomes then necessary to define a new model of interference adapted to this concept.

We denote $SNR_S(D)$ the Signal Noise Ratio at node D of a signal sent by node S. The minimum SNR to ensure error-free transmissions is β and $P(S,D)$ characterizes the received power at node D of signal sent by S. Classically, N parameter represents the ambient noise. Using PLNC, a node D can handle simultaneous reception of two signals from two different nodes S_1 and S_2 if the two following conditions are satisfied :

- $SNR_{S_1}(D) = \frac{P(S_1,D)}{\sum_{\substack{k\neq 1\\ k\neq 2}} P(S_k,D)+N} > \beta$
- $SNR_{S_2}(D) = \frac{P(S_2,D)}{\sum_{\substack{k\neq 1\\ k\neq 2}} P(S_k,D)+N} > \beta$

In this model, in order to decode at D the mixed signals from S_1 and S_2, both have to be individually received with a sufficient SNR. We assume here that the two signals are perfectly synchronized at the receiver, what constitutes a challenge in ad-hoc networks. However, mechanisms were already proposed to alleviate these stringent synchronization requirements [16] and real PLNC implementations made available [12] [13].

Unlike traditional packets receiving, it is important to keep in mind that in this model, the destination is not able to decode individually packets from nodes S_1 and S_2. When a PLNC transmission is performed, the receiver only retrieves the XOR-ed packet between the two sent messages. This constraint is a key point in the rest of the paper.

B. Notation and modeling parameters

In all the following, we model the wireless network as a graph $G = (N,E)$, where N and E represent respectively nodes set and edges set. A node n_i can send a packet to a node n_j if they are linked with a directed edge $e_{ij} \in E$. In our model, two nodes are linked and can then communicate if the distance between them is less than a certain threshold. This threshold represents the radio transmission range in realistic deployments.

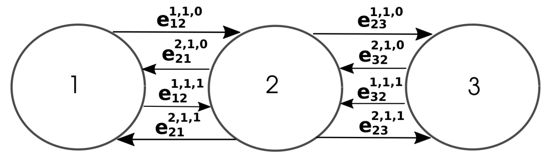


Fig. 2. New transmission graph in the TWRC scenario

We consider in the studied networks a set of unicast flows $\mathcal{F}$. Each flow $f \in \mathcal{F}$ is defined by four parameters :

1) A source $s(f)$.
2) A destination $d(f)$.
3) A set of paths between $s(f)$ and $d(f)$ denoted $\mathcal{P}(f)$.
4) A traffic demand $r(f)$.

We assume multi-path routing using potentially many routes between the source and the destination of each flow. We then assign to each link $e_{ij} \in E$ a "flow-path" identifier and we denote by $l_{ij}^{f,p}$ the amount of data of flow f, on link e_{ij} of path p. Note that $l_{ij}^{f,p}$ cannot exceed C_{ij}, the capacity of link e_{ij}. Obviously, for a link not belonging to path p of flow f, $l_{ij}^{f,p}$ equals zero. The traffic demand $r(f)$ represents the amount of data that a source wants to send for flow f. As for $l_{ij}^{f,p}$, this value is expressed in frame per slot.

In order to model realistic wireless conditions, we consider the physical model for traditional interference-free transmissions. As described earlier, this model takes into account the received power of a signal for computing the generated interference. We assume that a transmitted signal is attenuated with a constant factor $\frac{1}{d^\alpha}$, where d is the distance between the sender and the receiver and α is the path-loss exponent. In wireless communications, this path-loss exponent is typically comprised between 2 and 6. The exact value depends on propagation environment (obstructions, atmospheric conditions, ...). We take $\alpha = 3$ and $\alpha = 4$ to model free space areas. We assume that all senders transmit with the same power $P = 1$. Classically, we take $\beta = 2$ for the minimum Signal Noise Ratio.

IV. Basic Framework Description

We detail here our framework to precisely evaluate the achievable gains of Physical-Layer Network Coding in large ad-hoc networks. We formulate this problem as a constrained optimization problem. Solving the problem involves both selecting which nodes apply PLNC and scheduling the transmissions in a conflict free manner. For the sake of clarity, in this section, we apply the framework to the standard TWRC case. Other PLNC schemes are presented in later sections.

A. PLNC aware conflict graph

Scheduling the overall traffic involves identifying PLNC aware conflict graphs. This is illustrated in the TWRC scenario of Fig. 1. Nodes A and B exchange data through the same relay R and thus perform a PLNC transmission leading to the reception of a XOR-ed coded packet. To ensure that each end destination receives enough information to retrieve the native packets sent by its source, PLNC transmissions can not be applied on any packet. Typically, the broadcast transmission of coded packets from R to A and B (second step of a TWRC sequence) can not be involved in another PLNC transmission. This would give a packet encoded with an additional third packet. By imposing this scheduling rule, we ensure that each flow destination has enough information to retrieve native packets.

To take this into account, we define a new PLNC aware transmission graph. We therefore identify each link which is part of a TWRC scheme. Each TWRC element is characterized by the following characteristics:

1) Two pairs "flow-path"
2) A source node for the pair "flow-path" 1 (resp. "flow-path"2) which is the destination of "flow-path" 2 (resp."flow-path" 1)
3) A relay node

$\mathcal{T}$ is the set of TWRC elements of the network and $\mathcal{T}(f,p)$ denotes the TWRC elements for which the pair "flow-path" (f,p) is involved. Each link is then identified by a new parameter and the amount of data between n_i and n_j for (f,p) is now defined for each TWRC element. $l_{ij}^{f,p}$ becomes $l_{ij}^{f,p,T}$, where T is the TWRC identifier. If a transmission is not involved in a PLNC sequence (it respects the traditional physical model), we set $T=0$.

Fig. 2 shows the derived graph associated to the classical TWRC scenario. Flow 1 (resp. flow 2) represents the traffic between nodes 1 and 3 (resp. nodes 3 and 1). The 4 arrows with the assigned parameter $T=0$ identify interference free transmissions of native packets. The 4 other arrows designate transmissions involved in the two steps of the PLNC scheme of the TWRC element identified by $T=1$. Typically, the scheduling rule prevents links $e_{12}^{1,1,0}$ and $e_{32}^{2,1,0}$ to be active simultaneously because the corresponding transmissions have to respect the traditional physical model. However, $e_{12}^{1,1,1}$ and $e_{32}^{2,1,1}$ can be scheduled at the same time given that they belong to the same TWRC element.

The PLNC aware conflict graph is built as a classical conflict graph, but its vertices are given by the links of the new transmission graph G. It includes both traditional links ($T=0$) and links belonging to TWRC elements. The PLNC aware conflict graph is used to derive an achievable flow rate by finding all the different maximum sets of links I which can be scheduled simultaneously. For that, all vertices of the conflict graph are ranked in random order. A vertex to a "schedulable links set" is added if the associated link can coexist with all the links already in the set. A set of transmissions can coexist if each transmission satisfies the physical model or the PLNC transmission model, depending on its nature. We repeat the operation until all the maximum sets of links are found.

Notation	Description
$\mathcal{N}$	Set of nodes of the network
$\mathcal{F}$	Set of established flows
$\mathcal{P}(f)$	Set of paths of flow f
$s(f)$, $d(f)$	Source and destination of flow f
$r(f)$	Traffic demand of flow f (frame/slot)
λ	Reduction factor of the traffic demands
$\mathcal{T}$	Set of TWRC elements in the network
$\mathcal{T}(f,p)$	Set of TWRC elements implicating flow f and path p
T	TWRC identifier
$IN(T,n_i)$, $OUT(T,n_i)$, $RELAY(T,n_i)$	Boolean functions which returns 1 or 0 depending on node n_i is the IN, OUT, $RELAY$ of T or not
$e_{i,j}^{f,p,T}$	Link associated to the transmission of flow f, path p, involved in TWRC element T, from node n_i to node n_j
$l_{i,j}^{f,p,T}$	Amount of data through link $e_{i,j}^{f,p,T}$
C_{ij}^{T}	Maximum amount of data that node n_i can send to node n_j in the TWRC element T (frame/slot)
I_i	Set of links which can be scheduled simultaneously
K	Number of different sets I_i
q_i	Activity period of set I_i

TABLE I
SUMMARY OF USED NOTATIONS

B. Constrained optimization LP formulation

To compute the achievable gains of Physical-Layer Network Coding in large multi-hop networks, we formulate this problem as a constrained optimization problem. To ensure fairness between flows we compute the lowest reduction factor λ of the demands with which all of them can be satisfied.

In the sequel of this section we formulate the new constrained optimization problem and describe all the constraints it involves. Table I summarizes all the variables used in the formulation.

The function $IN(T,n_i)$ (resp. $OUT(T,n_i)$, resp. $RELAY(T,n_i)$) is the boolean function that returns 1 if node n_i is the source (resp. the destination node, resp. a relay) of TWRC element T, and returns 0 otherwise. Denoting I_i a set of links that can be scheduled simultaneously, q_i represents the fraction of time during which I_i is active. K is the number of discovered sets.

The first part of equality of constraint (1) represents the amount of data sent by each flow source along the different paths. We aim to maximize this value ensuring fairness. A source can transmit a packet to the first relay (or to the flow destination, if it is a one hop flow) following two different scenarios. First, a packet can be transmitted avoiding interference at the receiver ($T=0$). The amount of data sent this way is expressed in the first term of the sum ($l_{s(f),j}^{f,p,0}$). A packet can also be involved in a PLNC transmission of TWRC element T, if $s(f)$ is the source of this element. The

maximize λ
subject to

Constraint 1: Throughput computing based on injected packets

$$\forall f \in \mathcal{F}$$
$$\sum_{p\in\mathcal{P}(f)} l^{f,p,0}_{s(f),j} + \sum_{p\in\mathcal{P}(f)} \sum_{T\in\mathcal{T}(f,p)} IN(T,s(f)) \times l^{f,p,T}_{s(f),j} = \lambda \times r(f) \quad (1)$$

Constraint 2: Flow conservation for native packets

$$\forall f \in\mathcal{F}, \forall p \in \mathcal{P}(f), \forall n_i \in \mathcal{N}(f,p)\backslash \{s(f), d(f)\}$$
$$l^{f,p,0}_{i,j} + \sum_{T\in\mathcal{T}(f,p)} IN(T,n_i) \times l^{f,p,T}_{i,j} = l^{f,p,0}_{h,i} + \sum_{T\in\mathcal{T}(f,p)} OUT(T,n_i) \times l^{f,p,T}_{h,i} \quad (2)$$

Constraint 3: Flow conservation for coded packets

$$\forall f \in\mathcal{F}, \forall p \in \mathcal{P}(f), \forall T \in \mathcal{T}(f,p), \forall n_i \in \mathcal{N}(f,p)\backslash \{s(f), d(f)\}$$
$$RELAY(T,n_i) \times l^{f,p,T}_{i,j} = RELAY(T,n_i) \times l^{f,p,T}_{h,i} \quad (3)$$

Constraint 4: Bounds on the sent amount of data

$$\forall f \in\mathcal{F}, \forall p \in \mathcal{P}(f), \forall T \in \mathcal{T}(f,p) \cup 0, \forall n_i \in \mathcal{N}(f,p)$$
$$\begin{cases} l^{f,p,T}_{i,j} \leq C^T_{ij} \\ l^{f,p,T}_{i,j} \geq 0 \end{cases} \quad (4)$$

Constraint 5: Activity periods constraint

$$\sum_{i=1}^{K} q_i \leq 1 \quad (5)$$

Constraint 6: Sent amount of data constraint

$$\forall f \in\mathcal{F}, \forall p \in \mathcal{P}(f), \forall T \in \mathcal{T}(f,p) \cup 0, \forall n_i \in \mathcal{N}(f,p)$$
$$l^{f,p,T}_{i,j} \leq \sum_{l^{f,p,T}_{i,j} \in K_i} q_i C^T_{ij} \quad (6)$$

sum $\sum_{T\in\mathcal{T}(f,p)} IN(T,s(f)) \times l^{f,p,T}_{s(f),j}$ represents the amount of data involved in PLNC transmissions of all TWRC elements whose $s(f)$ is the source.

Constraint (2) is the constraint of flow-conservation for native packets of each flow. Each relay has to receive or decode the same number of native packets it transmits. There are two ways of decoding native packets. First, a node is the destination of traditional transmissions of native packets. By definition, all of them are interference free and the relay receives the native packet as it is sent by the previous hop. The received amount of data of traditional transmissions is $l^{f,p,0}_{h,i}$. Second, the relay can be the "Out node" of a TWRC element. With the scheduling rules detailed above, even if the received packet is coded (XOR between two native packets), we ensure that the relay has enough information to retrieve the intended native packet. Indeed, by authorizing only Physical-Layer Network Coding between links of the same TWRC element, we ensure that each TWRC "Out node" can retrieve its intended packet. The amount of coded data received and decoded, by node n_i, after the TWRC element T is $OUT(T,n_i) \times l^{f,p,T}_{h,i}$. Then, as in constraint (1), each decoded native packet has to be forwarded in a classical interference free transmission ($l^{f,p,0}_{i,j}$) or in a PLNC transmission in T, if the relay is the source of this TWRC element ($IN(T,n_i) \times l^{f,p,T}_{i,j}$).

Constraint (3) ensures flow-conservation for each coded packet, at each TWRC relay. Indeed, each TWRC relay has to forward every packet obtained from a PLNC reception (first step of TWRC sequence) to ensure that the two "Out nodes" receive and decode their intended packet. In brief, constraints (2) and (3) combined with scheduling rules, ensure the end-to-end flow conservation on one hand and that each flow destination decodes native packets sent by the source on the other hand.

Constraints (4), (5) and (6) are more classical. (4) ensures that the amount of data through each link is positive and lower than the capacity of the link. This capacity depends on the TWRC element in which the link is involved. Indeed, in PLNC sequences, broadcast communications are set up by the relay (following the PLNC reception). In these transmissions, the capacity is bounded by the lowest capacity of the two links involved. For instance, if nodes n_j and n_k are part of the same TWRC unit T with node n_i as a relay, we have $C^T_{ij} = C^T_{ik} = min(C_{ij}, C_{ik})$. Then, for each unicast transmission from node n_i to node n_j, $C^0_{ij} = C_{ij}$.

Each set I_i is active during a period q_i. Obviously, the sum of these fractions of time can not exceed 1 since sets are active sequentially one after the other (constraint (5)). Finally, the amount of data on each link $l^{f,p,T}_{i,j}$ is constrained by the fraction of time during which this link is active and its link capacity (constraint (6)). This value corresponds to the sum of periods during which a set of links I_i containing the link $l^{f,p,T}_{i,j}$ is active.

C. Flow rate upper bound

To derive an upper bound, we consider all the cliques of our conflict graph. A clique is subset of vertices such every two distinct vertices (corresponding to transmissions) can not be active in the same time. Denoting $\mathcal{C}$ the set of cliques and $q^{f,p,T}_{ij}$ the period of activity of link $e^{f,p,T}_{ij}$, as two physical links belonging to the same clique can not be active simultaneously, we obtain the following constraint :

Constraint 7: Cliques constraint

$$\forall C \in \mathcal{C}, \qquad \sum_{e^{f,p,T}_{ij} \in C} q^{f,p,T}_{ij} \leq 1 \quad (7)$$

Replacing constraints (4), (5) and (6) by this one, we derive an upper bound on the throughput. We tighten this bound by finding all the cliques of the conflict graph, with an algorithm similar to the one for finding independent sets.

V. Gain evaluation of TWRC Based Physical-Layer Network Coding

We first evaluate the potential gains of the TWRC Physical-Layer Network Coding in multi-hop networks. We compare

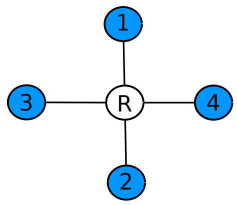


Fig. 3. Topology used for framework validation - 4 established flows (1→2, 2→1, 3→4 and 4→3)

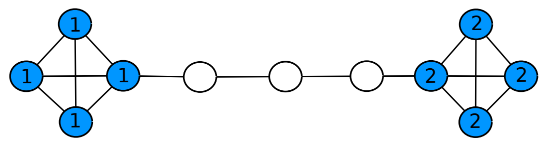


Fig. 4. Topology used to evaluate gains of PLNC in presence of multi-hop bi-directional/No constraint traffic

the maximum flow rate (throughput) obtained with traditional interference free transmissions (state of the art) to the maximum flow rate using PLNC transmissions. For both scenarios, we compute the maximum throughput by having the lower and upper throughput bounds converge on each other.

A. Framework validation

To validate our framework, we select a toy topology (Fig. 3) with two bi-directional flows between nodes 1,2 and 3,4, respectively. The maximum per flow throughput with traditional interference-free transmissions is 0.125 frame per slot. Only one node can be active at a time, meaning a source can only send a frame every 8 slots – an optimal flow-preserving scheduling assigns 4 slots to R and 1 slot to each of the four source nodes. With PLNC, each source node can send a frame every 4 slots. In slot 1, nodes 1 and 2 transmit simultaneously a packet to R. R broadcasts the received packets in slot 2. Similarly, in slot 3, nodes 3 and 4 can transmit simultaneously followed by R. It is straightforward to see that the maximum throughput is doubled with a (normalized) flow rate of 0.25 frame per slot. We applied our framework to this scenario and got the exact value of 0.25.

B. Gain evaluation in a multi-hop topology

The topology of Fig. 3, composed of 2 TWRC elements, undoubtedly benefits Physical-Layer Network Coding, explaining the factor 2 improvement. The difficult question, however, is, what gains are achievable in larger topologies with different traffic patterns. Hence, we investigate a new topology consisting of 16 nodes. 2 clusters (1 and 2) of 4 nodes each are connected by a set of 3 relays (Fig. 4).

We first simulate bi-directional traffic between the two clusters: the same number of flows between the two clusters is established. We evaluate the maximum throughput by varying the traffic load on the network (number of bi-directional flows established). For each number of flows, Fig. 5 shows the average of the obtained gains computed over 50 runs with randomly selected sets of established flows. The results

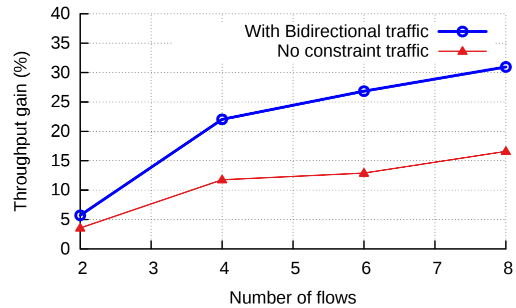


Fig. 5. Gains of TWRC PLNC in a multi-hop network depending on the traffic load

underline that the gains of PLNC are still significant in such a large topology. When traffic is high (more than two flows), the throughput gain is above 20%.

We now relax the constraints on the traffic patterns. Each of the 16 nodes of the topology can randomly select to be the source or the destination of a particular flow. In other words, the generated traffic is not necessarily bi-directional anymore. Unsurprisingly, Fig. 5 shows that PLNC gains are considerably reduced with these new conditions. This is due to the fact that, when bi-directional flows are not frequent, fewer Two-Way Relay Channel elements (a relay in the middle of two opposite flows) exist, hindering some of the PLNC benefits.

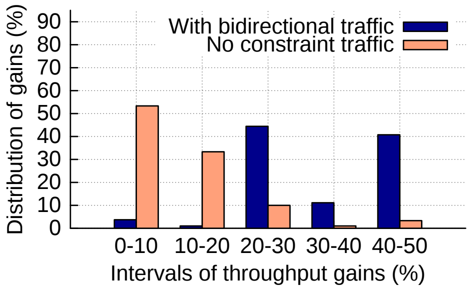
In Fig. 6(a) and 6(b), we present the gain distribution obtained for the 50 studied configurations for 4 and 8 established flows, respectively. For 4 flows, with bi-directional traffic, in 95% of the considered configurations, TWRC PLNC provides a throughput gain higher than 20%. However, less than 15% of the configurations without traffic constraint achieve these gains due to less PLNC opportunities. For 8 flows and bi-directional traffic, the obtained gains in the different scenarios are slightly higher and range between 30% and 40%.

VI. Advanced PLNC Schemes

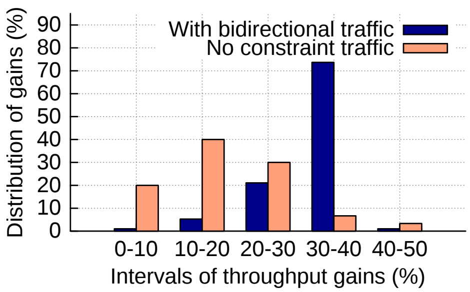
Previous results highlight the potential benefit of applying PLNC to large multi-hop networks. However, the traditional TWRC scheme does not seem to be sufficient when traffic is not perfectly bi-directional. In this section, we present two other approaches capable of fully taking advantage of the PLNC concept and evaluate their gains using our framework.

A. The Butterfly scheme

To benefit from the PLNC when traffic is not exactly bi-directional, a new way of setting up PLNC transmissions has been recently introduced in [15] and [16]. Called "Butterfly", the new scheme is a simplified version of the Butterfly introduced for the traditional network coding [3]. As illustrated in Fig. 7, 5 nodes are involved in this scenario: 2 sources, 2 destinations and 1 relay node. Each source (nodes 1 and 2) wants to send a packet to its destination (nodes 3 and 4) through the common relay (node R). Each destination is close enough to the other flow's source to overhear its packet. The opportunistic listening technique is then applied. Unlike



(a) 4 flows



(b) 8 flows

Fig. 6. Distribution of throughput gains depending on the traffic pattern for 4 and 8 established flows

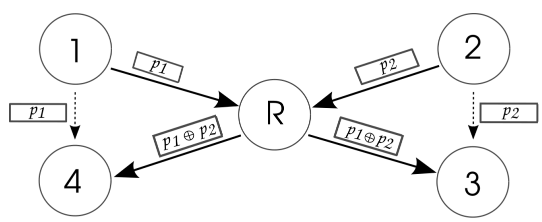


Fig. 7. The Butterfly PLNC (1→3 and 2→4) - 2 steps (time-slots) illustration

traditional protocols, where nodes capture only packets that are addressed to them, the destinations of the Butterfly scheme buffer all packets. In the first slot, sources simultaneously transmit their data to the relay, leading to a PLNC reception. In the second slot, the relay broadcasts the obtained XOR-ed packet to the two destinations. Thanks to the opportunistic listening in the first slot, each destination has enough information to retrieve its intended packet. In this scenario, the gain of the TWRC scheme is zero because there is no TWRC opportunity (no bi-directional traffic). With the Butterfly, on the other hand, the gain is 100% (2 slots instead of 4 with traditional communications).

Looking now at a larger topology with more concurrent unicast flows, one can see that estimating the gains with the Butterfly model becomes intricate. Indeed, the interference area of the Butterfly is larger than that of the TWRC scheme since more nodes are involved (5 instead of 3) what reduces the number of transmissions which can be scheduled simultaneously.

By replacing the TWRC elements (that are special cases of the Butterfly model) by Butterfly elements, our framework

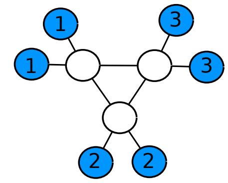


Fig. 8. Topology with 3 clusters for gains evaluation of Butterfly PLNC

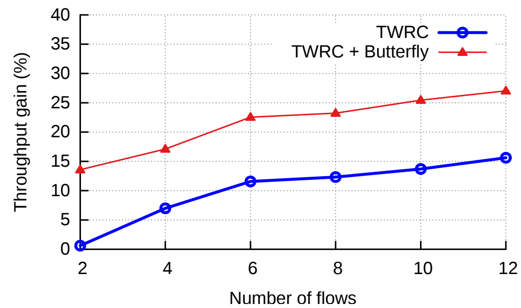


Fig. 9. Gain of Butterfly PLNC

can also be used to evaluate the gain the latter scheme. The scheduling rules here are slightly modified since transmissions in the first part of the PLNC sequence are now multicast (wireless broadcast). The received signal at each opportunistic listener also has to satisfy the physical model condition to ensure decoding. In addition, the linear program has to verify that the amount of data of new multicast transmissions is bounded by the lowest capacity of the two links.

We evaluate the gains of Butterfly PLNC on the topology shown in Fig. 8 in which nodes of 3 different clusters send data to other clusters through the relays. Results reported in Fig. 9 shows that Butterfly PLNC (that implicitly includes TWRC) outperforms with 2 times higher gains the TWRC schemes. Regardless of the traffic load, the Butterfly mode offers more opportunities for performing PLNC transmissions, leading to a throughput increase.

B. *Intra-flow PLNC*

Even if the Butterfly does not require exact bi-directional traffic as in the TWRC scheme, it is clearly inefficient when flows follow the same direction. A solution to take advantage of PLNC in such situations was already pointed in [2]. In a multi-hop flow, a relay knows packets sent by the next neighbor since it has already received them the step before. For example node 2 in Fig. 10, after a PLNC reception between packets $p1$ and $p2$ sent by the previous relay 1 and the following relay 3, can retrieve its intended packet $p2$. Indeed, it decodes the XOR-ed packet between $p1$ and $p2$ in the PLNC reception. It has already $p1$, it has enough information to retrieve $p2$. With that, in a chain topology, the flow rate is 0.5 frame per slot compared to 0.33 (1 frame every 3 slots) with traditional transmissions.

In contrast with TWRC and Butterfly schemes, this intra-flow PLNC does not require any particular traffic pattern to

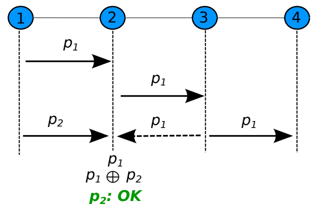


Fig. 10. Intra-flow PLNC concept

be efficient, but surprisingly it was not studied in large ad-hoc network.

We adapt our previous framework to evaluate potential gains of intra-flow PLNC in large multi-hop networks. Indeed, intra-flow PLNC is based on the assumption that each relay already knows the packets sent by the following node. However, this condition is not always satisfied in a multi-hop network especially when combined to the TWRC scheme. In particular, the relay node of TWRC elements does not know the native packets sent by the two sources. It just decodes and relays the XOR-ed packet. Thus, in the next slots, this relay can not perform intra-flow PLNC with a second packet of the same flow since it does not have enough information to retrieve the second packet.

To account for this new condition, we assign a new boolean parameter a to each link of the interference graph. We enforce $a = 1$ or $a = 0$ depending on whether the corresponding packet transmission follows (comes immediately after) a TWRC element or not respectively. The amount of data through each link is then denoted $l_{i,j}^{f,p,T,a}$. Preventing a link with $a = 1$ to be involved in an intra-flow PLNC transmission at its previous node, we ensure that each node can decode its intended packets after each intra-flow PLNC reception. To preserve flow conservation, we derive a new linear program extending the previous one. Constraints (1), (4), (5) and (6) remain unchanged. However, constraints (2) and (3) are replaced by constraints (8), (9) and (10).

Constraint (8) ensures conservation of each packet received in the broadcast transmission of a TWRC sequence. Indeed, with our definition, all of them are relayed in what we identify as transmissions following a TWRC sequence ($a = 1$). Intra-flow PLNC can not be processed with these transmissions.

Constraint (9) ensures flow conservation for packets received in interference free transmissions. These transmissions can follow a TWRC sequence or not ($a = 0/1$). They can be forwarded by each relay in another interference free transmission or in a TWRC sequence (if the concerned node is the source of a TWRC element).

Constraint (10) is similar to constraint (3) of the previous linear program. The only difference is that the relay of a TWRC element can receive packets identified as following another TWRC sequence (i.e with $a = 0/1$).

By solving the new optimization problem, we illustrate potential gains of intra-flow PLNC in the presence of unidirectional flows (Fig. 11). In this topology, up to 6 sources have data to send to a common destination D. Results depicted

Constraint 8: Flow conservation for packets following a TWRC sequence

$$\forall f \in \mathcal{F}, \forall p \in \mathcal{P}(f), \forall n_i \in \mathcal{N}(f,p) \setminus \{s(f), d(f)\}$$
$$\sum_{T \in \mathcal{T}(f,p)} IN(T, n_i) \times l_{i,j}^{f,p,T,1} + l_{i,j}^{f,p,0,1} = \sum_{T \in \mathcal{T}(f,p)} OUT(T, n_i) \times l_{h,i}^{f,p,T,0} \quad (8)$$

Constraint 9: Flow conservation for native packets

$$\forall f \in \mathcal{F}, \forall p \in \mathcal{P}(f), \forall n_i \in \mathcal{N}(f,p) \setminus \{s(f), d(f)\}$$
$$l_{i,j}^{f,p,0,0} + \sum_{T \in \mathcal{T}(f,p)} IN(T, n_i) \times l_{i,j}^{f,p,T,0} = l_{h,i}^{f,p,0,0/1} \quad (9)$$

Constraint 10: Flow conservation for coded packets

$$\forall f \in \mathcal{F}, \forall p \in \mathcal{P}(f), \forall T \in \mathcal{T}(f,p),$$
$$\forall n_i \in \mathcal{N}(f,p) \setminus \{s(f), d(f)\}$$
$$RELAY(T, n_i) \times l_{i,j}^{f,p,T,0} = RELAY(T, n_i) \times l_{h,i}^{f,p,T,0/1} \quad (10)$$

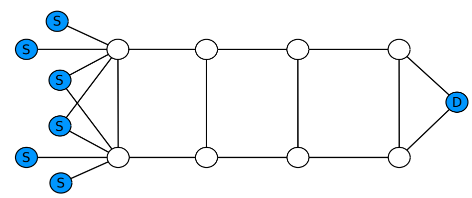


Fig. 11. Studied topology to evaluate gains of intra-flow PLNC in presence of multi-hop unidirectionnal flows

in Fig. 12 highlight that whatever the traffic load (number of active sources), the throughput is increased of almost 50% with intra-flow PLNC. Of course, the TWRC and Butterfly schemes do not provide any gain in this context since no TWRC neither Butterfly elements can be identified here.

C. Generalization of Previous Results

Previous results showed that PLNC performance closely depends on studied topology and traffic patterns. One can wonder whether the 3 studied PLNC schemes still fairly perform in realistic environments.

For that, we pick a random topology of 16 nodes depicted in Fig. 13 and modify traffic load in order to evaluate the gains of different PLNC schemes. For each flow, a random source and destination are selected. Moreover, for every traffic load, 20 runs are realized with different randomly selected source and destination for each run. We compare using our framework the 3 PLNC schemes and their combination.

We show in Fig. 14 the average gain obtained for different number of flows. As in all previous results a flow is identified by a source, a destination, a traffic demand and the possible routes. As expected, the TWRC scheme, if used alone, suffers from the lack of PLNC opportunities, even if gains reach 15% with 15 established flows. Besides, Butterfly scheme

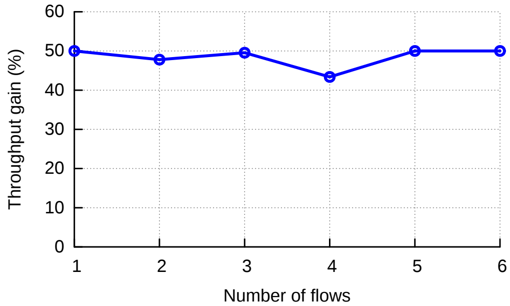


Fig. 12. Gains of intra-flow PLNC in presence of multi-hop unidirectionnal flows

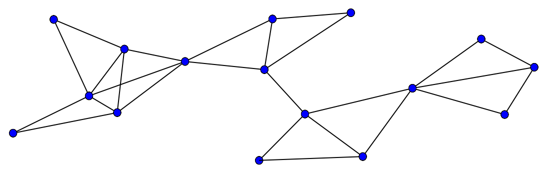


Fig. 13. Realistic topology

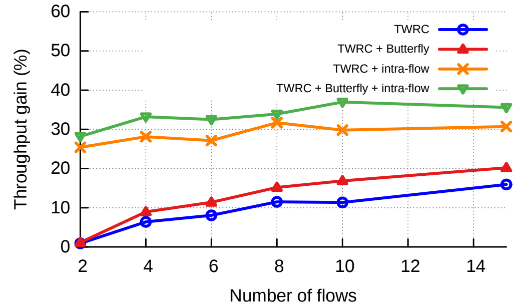


Fig. 14. Gains of the different PLNC schemes in realistic environments

(including implicitly TWRC) outperforms TWRC for any traffic load. More interestingly, the intra-flow PLNC addition the two other techniques offers a stable gain of around 30% regardless of the considered number of flow. In fact, the intra-flow PLNC is less dependent on the traffic pattern and can be applied in almost all multi-hop topologies. Finally, the three derived schemes are not incompatible. The best solution would be to integrate all of them in large multi-hop networks, as highlighted in Fig. 14.

VII. Conclusion and Future Work

In the coming years, wireless networks are expected to experience an exponential growth in the number of devices, severely challenging the access techniques known today. Physical-Layer Network Coding (PLNC) can be seen as a candidate technique to increase the efficiency and reduce the interference in these dense networks. In this work, we have investigated the capability of PLNC to enhance throughput in large multi-hop networks. To do so, we have formalized the scheduling problem in multi-hop wireless network with PLNC as an optimization problem. We have introduced a novel interference graph that captures how PLNC changes the definition of interference in multi-hop wireless networks. Starting with the widely used two-way relay channel model, we have computed the achievable throughput in large topologies. Then, we have empowered our framework with the capability of accounting for additional and more complex PLNC schemes such as the Butterfly and the intra-flow models. Our results show that, with these models, PLNC increases throughput by 30% in realistic topologies.

In the future, we aim to enrich our framework with other scheduling policies. For now, our framework has adapted optimal flow-preserving scheduling to be implemented with TDMA-based access. What will the performance gains be with other scheduling policies such as that produced by the IEEE 802.11 MAC? Furthermore, we plan to implement and validate our result in a testbed based on USRP devices and the GNU radio framework.

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