On Estimating the End-to-End Bandwidth in Multi-Transceiver Multi-Hop Cognitive Radio Networks

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ABSTRACT

Cognitive radios promise to revolutionize the performance of wireless networks in general and multi-hop wireless networks in particular by making efficient use of the portion of the licensed spectrum left un-utilized. Realizing this promise, however, requires revisiting many of the current network architectures and protocols, which is the subject of a very active research effort. In this work, we focus on Quality of Service routing and more specifically, admission control. We consider a multi-hop cognitive radio network where every node is equipped with multiple transceivers. Because the research and development of a widely accepted MAC protocol for these networks is still ongoing, we assume a bare-bones TDMA protocol at the link layer. We show that, for the network considered, the problem of finding the maximum end-to-end bandwidth of a given path is NP-Complete. Given this result, we consider a relaxed version of the problem wherein the slot allocations are carried out at each node by selecting at random the required number of slots among those available. For this case, we provide a linear time algorithm for computing the average residual end-to-end bandwidth. We perform an extensive numerical analysis that demonstrates its accuracy and enabling value for performing admission control.

Categories and Subject Descriptors

C.4 [**Performance of Systems**]: Modeling techniques; D.2.8 [**Software Engineering**]: Metrics—*complexity measures, performance measures*

Keywords

Multi-hop Cognitive Radio Networks; Bandwidth Estimation; TDMA

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1. INTRODUCTION

Traditionally, the wireless spectrum has been treated as a scarce commodity that has been tightly regulated by centralized authorities, such as the FCC in the United States. Operating on wireless spectrum requires either applying for a license by the respective governmental authority or, requires using select frequencies, like those in the ISM band, that are freely available to everyone. The latter approach is followed by many popular wireless technologies, such as IEEE 802.11, Bluetooth, sensor networks, etc, which has led to the ISM frequencies often being overcrowded. At the same time, extensive measurements have shown that a significant part of the licensed spectrum remains unutilized [19]. As a result, the FCC issued a historic ruling permitting unlicensed devices to use unutilized licensed spectrum [1]. This ruling, coupled with the emergence of the cognitive radio network concept [16], have ignited a lot of interest in the research and development of cognitive radio networks capable of exploiting the best spectrum available [19]. While the concept of cognitive radio networks holds great promise, a lot of technical and policy challenges need to be ironed out before the concept can turn into reality [4]. Significant effort has been focused at the challenges arising at the physical, MAC and network layers [10, 4]. The IEEE 802 standards committee has chartered the IEEE 802.22 working group to write a standard aimed at using cognitive radios to allow sharing of the unused spectrum allocated to the Television Broadcast Service for broadband Internet access [2]. The primary focus so far have been the single-hop architectures [2]. However, multi-hop wireless networks could greatly benefit from using cognitive radios [3]. To this end, novel algorithms for routing and spectrum allocations have been proposed in the literature. However, the field is in a nascent stage and so far the focus has not been on the quality of service routing. With applications like high-definition video and voice becoming prominent on the Internet and spilling over to wireless devices, it is crucial that any new architecture provide support for quality of service.

In this work, we focus on quality of service provisioning in multi-hop cognitive radio networks wherein every node is equipped with multiple transceivers [18]. Our goal is to provide admission control at the routing layer, therefore, the main problem considered is that of computing the residual end-to-end capacity of a given path. Because the field is in its early stages and highly evolving we do not tailor our solution to a specific routing and MAC protocol to avoid becoming obsolete in a short time. Rather, we only assume the routing to be cognitively adapted and consider a bare-bones TDMA protocol implemented at the MAC layer. With channel sensing, which usually requires periodic channel allocations, being a requirement for cognitive radio networks [4], we believe a TDMA-like protocol is the most likely technology to be adopted at the link layer [2, 10, 9]. For the network considered, we prove that computing the maximum end-to-end bandwidth is NP-Complete. Given this result, we consider a relaxed version of the problem wherein for a given request, the necessary TDMA slots at every link are allocated by selecting them at random among those available. For this case, we design a linear time and centralized algorithm for estimating the end-to-end bandwidth of a given path. To evaluate the performance of our algorithm, we perform an extensive numerical analysis in MATLAB. Our analysis demonstrates the accuracy of our algorithm in computing the end-to-end bandwidth for a variety of path lengths, as well as, its capability in providing correct information for performing admission control.

The rest of the paper is organized as follows. In Section 2, we discuss some related work. In Section 3, we describe in detail the system model. In Section 4, we formally define the problem of computing the end-to-end bandwidth in TDMA-based multi-transceiver multi-hop cognitive radio networks wherein the required slots are selected randomly among those available. In Section 5, we give the centralized solution to the bandwidth calculation problem while in Section 6 we discuss the performance evaluation. Finally, we conclude the paper in Section 7.

2. RELATED WORK

Multi-hop cognitive radio networks share similarities with the traditional multi-hop networks, chiefly among them the wireless interference. At the same time, cognitive radio networks present unique challenges, most importantly the presence of the so-called primary users and a far more rich and dynamic selection of channels. This has resulted in new architectures and protocols being proposed for cognitive radio networks. Nevertheless, considering the similarities between the two architectures, a lot of the research results for traditional multi-hop wireless networks are relevant and in many cases have been at the basis of the newly proposed protocols.

Works in Cognitive Radio networks: A main goal of any cognitive radio architecture is to protect the primary users from interference [7]. At the routing layer, a node is required to adapt its path computations according to the primary users activity. To this end, it can either route around the primary users, thus potentially increasing the path length, or, switch its transmission channel on the affected links. Obviously, both strategies will increase the end-to-end delay. In [8] is proposed a geographic routing solution that selects next hops and operating channels so as to avoid regions of primary users activity while minimizing the end-to-end path latency. Joint route selection and spectrum decision is also addressed in [22, 6]. In [22] the authors propose establishing a spectrum tree on each frequency channel and selecting routes according to a newly defined routing metric. An optimal routing metric for multi-hop cognitive radio networks is proposed in [6]. The authors analytically demonstrate its optimality and accuracy for the cases of mobile and static networks. While the works presented so far are shown to handle well the primary users, none of them addresses the problem of admission control for quality of service. Works closer to the problem considered in our work can be found in [20, 17, 14] wherein algorithms for joint routing, link scheduling and spectrum assignment have been studied. In [20] an opportunistic scheduling that maximizes the overall capacity of secondary users while satisfying a constraint on time average collision rate at the primary users is proposed. In [17] the joint routing and link scheduling problem with uncertain spectrum supply is investigated. The authors in [14] address the problem of minimizing the total transmission latency. Finally, [11] has proposed a distributed algorithm for jointly optimizing routing, scheduling, spectrum allocation and transmit power. Nevertheless, the problem of computing the available end-to-end bandwidth of a multi-hop path is not addressed in any of these works.

Works in non-cognitive radio architectures: The problem of QoS in non-cognitive wireless multi-hop architectures, with a single or multiple radios, has been subject of significant research efforts and an exhaustive survey is beyond the scope of this paper. Instead, here we simply summarize a subset of the published works that is closest to the work presented in this paper. The problem of admission control for QoS routing in multi-hop networks is studied by numerous works, including [21] and the references therein. In [21], it is shown that for a TDMA architecture, the problem of computing the residual end-to-end bandwidth for a multi-hop path is NP-Complete. Intuitively speaking, the problem is hard because computing the residual end-to-end bandwidth is coupled with the problem of per-link slot assignments. With the problem being NP-Complete, a greedy heuristic is proposed and incorporated in the AODV routing protocol. However, this heuristic was designed for a single non-cognitive radio architecture and cannot be readily applied to a cognitive radio architecture. In [12] the authors consider the problem of joint routing and link scheduling and propose a solution which consists of forming a set of constraints and solving a linear programming problem. A set of necessary conditions for a rate vector to be achieved is available in [13]. However, in these works the authors try to determine the maximum amount of flow that can be sent from a given source to a given destination in a capacitated network. Instead, in our work, we focus on the achievable rate of a given path. In [5], the authors study the joint routing and channel assignment problem for the case of infrastructure wireless mesh networks with multiple radios. They propose a constant approximation algorithm to the NP-hard problem of maximizing the overall network throughput, subjected to fairness constraints. Similarly, [15] provides a distributed, online and provably efficient algorithms for joint routing, channel assignment and scheduling in multi-hop multi-radio networks. These algorithms guarantee a fraction of the maximum achievable system capacity.

3. SYSTEM MODEL

We model a multi-hop cognitive radio network as a graph $G = (V, E)$, where V is the set of cognitive radio nodes and E the logical data links, all assumed to be symmetric. Every cognitive radio node is equipped with a constant number of half-duplex transceivers, each capable of sensing and transmitting on B predefined orthogonal wireless channels. In this way, every node can potentially transmit and receive data simultaneously on different channels. An additional transceiver can be used for control signaling. We assume that the channel assignments are done by one of the joint routing and spectrum allocation process presented in section 2. This step is beyond the scope of this work as our main concern is about estimating the available end-to-end bandwidth at the MAC layer. We simply address the case of every node having multiple transceivers but only one frequency channel being assigned between a pair of nodes. Thus, we model a path as a directed chain $n_1 \rightarrow n_2 \cdots \rightarrow n_{N_H+1}$ wherein each of the N_H links, also referred to as hops, operates on one channel. This channel can either be the same along the path or be different for a pair of links. This actually depends on how the best path is selected at the routing layer. Every link offers a specific data rate that depends on both the selected channel and the current environment conditions. To schedule data transmissions, a TDMA MAC with frame size S and constant time-slot duration is finally implemented on every assigned channel. For a given path, when a node n_i needs to transmit data to node n_{i+1}, it can access the medium in a contention-free manner by reserving time-slots on the corresponding channel. For ease of presentation, we refer to a pair $(channel, timeslot)$ simply as a slot.

4. PROBLEM DEFINITION

In computing the end-to-end bandwidth of a multi-hop path in a cognitive radio network, one needs to take into account two sources of interference: the interference from the primary users and that from the other secondary users operating on the same channel. When a primary user shows up, it is detected by the sensing and the spectrum decision module hence leading to a channel reassignment[1]. Should a node n_i need to reserve a new time-slot to transmit data to n_{i+1}, it does so on the corresponding assigned channel. However, due to the potential interference from other cognitive radios operating on the same channel, for the time-slot to be selected, it needs to satisfy the following requirements:

1. This time-slot is not used on this channel by node n_i for transmitting,
2. This time-slot is not used on this channel by any 1-hop neighbor of node n_i for transmitting,
3. This time-slot is not used on this channel by any 2-hop neighbor of node n_i for transmitting.

The purpose of the first condition is obvious. The second one implies that n_i cannot transmit in a time-slot during which it is supposed to listen to data transmitted by one of its 1-hop neighbors. The last one prevents node n_i from transmitting during a time slot during which one of its 1-hop neighbor is supposed to listen to another transmission, thus avoiding collision at this neighboring node. We assume every node knows the slot allocations in its two-hop neighborhood[2] and thus can check the satisfiability of the above constraints.

[1]The exact sensing and spectrum allocation mechanism is orthogonal to our solution and beyond the scope of this work.

[2]This information can be easily obtained by sending beacons containing bitmaps with slots scheduled for transmission or reception for the node itself and its one-hop neighbors.

From now, slots satisfying these requirements are considered as available.

THEOREM 1. *The problem of computing the maximum end-to-end bandwidth of a given path in TDMA-based multi-hop cognitive radio networks with multiple transceivers is NP-complete.*

PROOF. Due to space limitations we provide a sketch of the proof. We show that our problem is NP-Complete by reducing the problem of computing the maximum path bandwidth in a single-channel TDMA-based multi-hop network, therein referred to as P_2, to our problem, therein referred to as P_1. To this end, we consider the instance of P_1 where a same channel with a constant data rate is assigned on every link along the path. Solving P_1 actually consists of solving one instance of P_2. Since P_2 has been shown to be NP-complete in [21], that concludes our proof. □

The optimal slot scheduling problem being NP-complete, we propose to work on a relaxation of this problem by addressing the case of random slot allocation at the link layer. To the best of our knowledge, the related bandwidth calculation problem remains open. Let the demand d, expressed in bits per second, refer to the amount of bandwidth required end-to-end by an application.

Achievable end-to-end rate: For a given path connecting a source to a destination, the first question we would like to answer is whether this demand can be satisfied end-to-end. As stated previously, a path is modeled as a directed chain $n_1 \rightarrow n_2 \cdots \rightarrow n_{N_H+1}$ composed of N_H hops. We analyze the network behavior when admitting a new incoming flow with demand d. Let a_i denote the additional number of slots allocated on every hop $i \in \{1, .., N_H\}$ for servicing this flow. Going forward we use the exponential notation (j) to specify that the considered quantity is evaluated just before node n_j does its allocations. We also define $A_i^{(j)}$ as the number of slots available at node n_i for communication on the link $n_i \rightarrow n_{i+1}$ on the corresponding channel just before n_j does its allocations for servicing this flow. For ease of presentation, we denote a link $n_i \rightarrow n_{i+1}$ as l_i. Assuming every node n_i knows the link l_i transmission rate, referred to as ϕ_i, n_1 can convert the flow demand d to the required number of additional slots to be allocated on l_1 as follows: $r_1 = \left\lceil \frac{d \times S}{\phi_1} \right\rceil$. This means that exactly $a_1 = \min(r_1, A_1^{(1)})$ slots will be reserved on the first hop. As a consequence a resulting traffic flow with bit-rate equal to $d_1 = \min\,(d, a_1 \times \phi_1/S)$ bits per second will be relayed on l_1. Note that in the network considered, the reservations are done locally and not end-to-end. Therefore, n_2 will intend to relay this new incoming traffic flow by using the same mechanisms and considering a flow demand d_1. We can thus easily infer that for any hop $i > 1$:

$$r_i = \left\lceil \frac{d_{i-1} \times S}{\phi_i} \right\rceil \tag{1}$$

$$a_i = \min\,(r_i, A_i^{(i)}) \tag{2}$$

and

$$d_i = \min\,(d_{i-1}, a_i \times \phi_i/S) \tag{3}$$

These formulae illustrate the close mathematical relation between the quantities a_i and $A_i^{(i)}$. Because on a given channel a node cannot reserve a slot used for transmission by one of its 1-hop or 2-hop neighbors, $A_i^{(i)}$ is likely to decrease if allocations are done on links l_{i-2} and l_{i-1}. Therefore, the number of slots allocated on every hop highly depends on the demand as well as the channels selected along the path. For instance when l_1 and l_2 operate on the same channel and all the slots are available for reservation on the first hop, a demand requiring the reservation of all these slots would leave zero slot available for communication on the second link and the resulting end-to-end bandwidth would be zero. Thus, for a specific demand d, the achievable end-to-end rate equals $\min(d_1, d_2, ..., d_{N_H})$. As $d_i \geq d_{i+1}$, this quantity finally equals d_{N_H}.

End-to-end capacity: The residual end-to-end capacity corresponds to the maximum achievable bandwidth between a source and a destination. As the achievable end-to-end throughput depends on the demand, this capacity is equivalent to the maximum end-to-end rate obtained for an arbitrary demand. Mathematically speaking, this metric can be calculated by solving the following problem:

$$b_{\max} = \max_{d \in I_d} \quad d_{N_H}(d) \tag{4}$$

with $I_d = [0, \min(\phi_1, \phi_2, ..., \phi_{N_H})]$. We relax this problem by evaluating the function d_{N_H} with a finite number of values d taken in the range I_d with step Δ_ϕ. Basically, Δ_ϕ can be taken equal to $\min(\phi_1, \phi_2, ..., \phi_{N_H})/S$. Therefore, as S is constant, the related calculation complexity only depends on N_H, that is, the path length.

5. CENTRALIZED SOLUTION

In the following, we propose an analytical framework for evaluating the average throughput that would be achieved on every link of a path if a new flow with demand d were to be admitted. This solution is centralized in that, the source node is assumed to have global knowledge of the network. For the sake of clarity, we define a new data structure indicating, for every link of a path, which slots are available for reservation. We call this structure the route slot availability table (RSAT). This table is composed of N_H lines and $S \times B$ columns. Each line i, composed of B sub-blocks of size S, indicates the available slots on link l_i. Then, each sub-block refers to the available time-slots on each sensed frequency channel. All the time-slots of each of the $B-1$ frequency channels not selected for communication on link l_i, are considered unavailable and the corresponding entries in the table are set to 0. A simple example of an RSAT table is depicted in Figure 1. Whenever at a node the spectrum decision module carries out channel reassignment, the RSAT is updated accordingly. Therefore, the RSAT summarizes all the necessary information required for computing the average number of slots that would be allocated on every hop of the path if a new flow with demand d were to be admitted. At the same time, by using equations 1, 2 and 3, it can derive the corresponding achievable rates.

5.1 Fundamental principles of the method

For practicability, we relax the problem by working on average values. Even though mathematically speaking $\mathrm{E}[a_3] \neq \min\left(\mathrm{E}[r_3], \mathrm{E}[A_3^{(3)}]\right)$, we do such an approximation of the average number of slots allocated on the third hop to reduce the calculation complexity. As depicted in section 6,

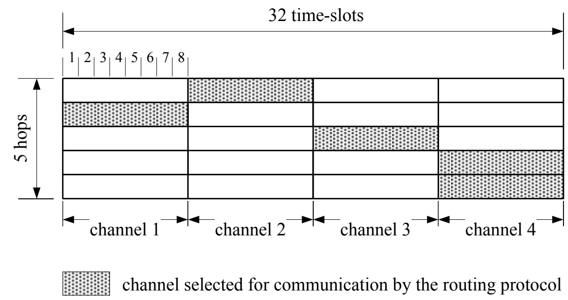

Figure 1: Route slot availability table for a 5-hop path. In this example, every link is assigned a channel among the four sensed by the spectrum decision module. Each TDMA frame is composed of eight time-slots.

our simulation results show that finally, this approximation does not degrade the performance of the overall estimation process. Based on this assumption, a first solution consists of estimating for each hop i the quantity $A_i^{(i)}$ which is now considered an average for the remaining of the paper.

l-link available slot set decomposition: From the RSAT, we can calculate for any communication link l_i the set $S_i^{(1)}$ containing the index of slots available for reservation on that link at the beginning of the estimation process. The indexes of slots are now taken in the set $\{1, 2, .., S.B\}$ related to the super-frame composing the whole line in the RSAT. It is also crucial to see that, when considering such an indexation, when a slot is allocated on l_i, it cannot be allocated anymore on both l_{i+1} and l_{i+2} as this would create interference. Thus, when this slot is only available for reservation on link l_i and neither on links l_{i+1} nor l_{i+2}, the sets $A_{i+1}^{(j)}$ and $A_{i+2}^{(j)}$ are not impacted. Inversely, if this slot is also available to one of these links, the corresponding sets are impacted and thus the number of slots that would be allocated on the next hops is likely to decrease. Thus, each slot belongs to a certain category depending on the links it appears to be available for reservation to. Therefore, we propose to divide the set $\{1, ..., S.B\}$ in non overlapping subsets that cover $\{1, ..., S.B\}$ and permit to categorise every slot according to the links it is available for reservation to in the RSAT table. To be more precise, we define such a decomposition on a set of l consecutive links $\{i, i+1, ..., i+l-1\}$ along the path. For $k \in \{0, 1, ..., l\}$, the number of subsets characterizing the slots available to a set of k links but not the $l-k$ others is $\binom{l}{k}$. Thus, the total number of subsets in the decomposition is $\sum_{k=0}^{l} \binom{l}{k} = 2^l$. We refer to such a decomposition as a *l-link available slot set decomposition* and use the following notations $E_{a,\bar{b},c}^{(j)}$ to denote the set of slots available for reservation on both two links a and c but not b just before node n_j does its allocations. Its cardinality is written as $C_{a,\bar{b},c}^{(j)}$.

To elucidate the meaning of these variables, let us consider a 3-hop path with the first channel selected for communication on every hop, $B = 2$, $S = 8$ and $S_1^{(1)} = \{2, 3, 4, 5\}$, $S_2^{(1)} = \{2, 6, 7\}$ and $S_3^{(1)} = \{1, 2, 4, 5, 6, 7\}$. This leads to the following eigth sets: $E_{1,2,3}^{(1)} = \{2\}$, $E_{1,2,\bar{3}}^{(1)} = \emptyset$, $E_{1,\bar{2},3}^{(1)} = \{4, 5\}$, $E_{\bar{1},2,3}^{(1)} = \{6, 7\}$, $E_{1,\bar{2},\bar{3}}^{(1)} = \{3\}$, $E_{\bar{1},2,\bar{3}}^{(1)} = \emptyset$, $E_{\bar{1},\bar{2},3}^{(1)} = \{1\}$ and $E_{\bar{1},\bar{2},\bar{3}}^{(1)} = \{8, 9, ..., 16\}$.

Achievable end-to-end throughput calculation: The random nature of the slot allocation process provides good properties to evaluate the average number of slots impacted in every subset as further depicted for the case of a 3-hop path for which a flow with demand d needs to be relayed from the source to the destination. From now on, we work with average values. From the RSAT, we can compute the 3-link available slot set decomposition related to l_1, l_2 and l_3. At the same time, the initial number of available slots on every communication link $A_i^{(1)}$ can be calculated. To forward the new traffic flow to its next hop n_2, node n_1 reserves exactly $a_1 = \min(r_1, A_1^{(1)})$ additional slots on the first communication link. Among these slots, some might have also been available for reservation on links l_2 and l_3 but, due to interference, become unavailable after these allocations.

Let us consider a discrete random variable X_i taking its values in the set $\{0, 1, ..., a_i\}$ and representing the number of slots initially available for reservation on link l_i that have been reserved by node n_1 to relay the new incoming flow on l_1. X_i represents the number of slots in the set $S_1^{(1)} \cap S_i^{(1)}$ reserved by n_1 for communication on l_1. Intuitively, as the slots are allocated at random, we see that a proportion $a_1/A_1^{(1)}$ of these slots are likely to be reserved by n_1. Mathematically speaking, X_i follows an hypergeometric distribution with parameters $(A_1^{(1)}, |S_1^{(1)} \cap S_i^{(1)}|, a_1)$. The expectation value of such a random variable is $\mathrm{E}[X_i] = |S_1^{(1)} \cap S_i^{(1)}| \times a_1/A_1^{(1)}$ and thus, in every set $S_1^{(1)} \cap S_i^{(1)}$, an average proportion $p_1 = a_1/A_1^{(1)}$ of slots is reserved by node n_1. Note that for the case of $A_1^{(1)} = 0$ we get $a_1 = 0$ and $p_1 = 0$. Exactly the same analysis can be carried out on every set resulting from the 3-link available slot set decomposition related to l_1, l_2 and l_3. This way, the average values $A_2^{(2)}$ and $A_3^{(2)}$ just after n_1 did its reservations can be computed as detailed in algorithm 1. Therefore, the average number of slots allocated on the following links can be evaluated and the process repeated until the average number of slots allocated on the last hop is calculated.

This approach still applies when increasing the path length. However, the impact of n_1 allocations is still required to be evaluated when computing the average number of slots that would be allocated on any further communication link l_i. Such an impact can be evaluated by first doing the i-link available slot set decomposition and then carefully measuring the dependence of each previous node allocations. This leads to an exponential number of sets to deal with which makes the solution not tractable. We refer to this phenomenon as the domino effect. To address this issue, in the following we introduce the clique sliding approach that breaks the domino effect and reduces the calculation complexity from exponential to linear.

5.2 Clique sliding approach

The clique sliding approach breaks the domino effect by processing the end-to-end bandwidth estimation clique by clique while using only a linear number of variables. The basic idea consists of eliminating the dependence on the previous node allocations. We define a clique as any set of three consecutive links on the path. For instance, a 4-hop path is composed of two cliques: $c_1 = \{l_1, l_2, l_3\}$ and $c_2 = \{l_2, l_3, l_4\}$.

Initialization: Given a path of length N_H, we start by computing the available slot sets resulting from the 3-link available slot set decomposition of every clique. This leads to eight corresponding sets for each clique, that is for the i^{th}: $E_{\overline{i},\overline{i+1},\overline{i+2}}$, $E_{i,\overline{i+1},\overline{i+2}}$, $E_{\overline{i},i+1,\overline{i+2}}$, $E_{\overline{i},\overline{i+1},i+2}$, $E_{i,i+1,\overline{i+2}}$, $E_{i,\overline{i+1},i+2}$, $E_{\overline{i},i+1,i+2}$ and $E_{i,i+1,i+2}$. The following describes how to extend the bandwidth estimation process when sequentially passing the calculation on to the next cliques.

Algorithm 1: Estimation of the average achievable end-to-end rate on a 3-hop path.

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input : d, S, ϕ_1, ϕ_2, ϕ_3, $S_1^{(1)}$, $S_2^{(1)}$, $S_3^{(1)}$
output: a_1,a_2,a_3,d_1,d_2,d_3
begin
  /* Initialization */
  $\forall i \in \{1, 2, 3\}, \quad A_i^{(1)} \leftarrow |S_i^{(1)}|;$
  /* Available slot set decomposition */
  $C^{(1)}_{\overline{1},\overline{2},\overline{3}}, C^{(1)}_{1,\overline{2},\overline{3}}, C^{(1)}_{\overline{1},2,\overline{3}}, C^{(1)}_{\overline{1},\overline{2},3}, C^{(1)}_{1,2,\overline{3}}, C^{(1)}_{1,\overline{2},3}, C^{(1)}_{\overline{1},2,3}, C^{(1)}_{1,2,3};$
  /* Allocations on l_1 */
  $r_1 \leftarrow \lceil d \times S/\phi_1 \rceil;$
  $a_1 \leftarrow \min(r_1, A_1^{(1)});$
  $d_1 \leftarrow \min(d, a_1 \times \phi_1/S);$
  $p_1 \leftarrow a_1/A_1^{(1)};$
  $A_2^{(2)} \leftarrow A_2^{(1)} - p_1.(C^{(1)}_{1,2,\overline{3}} + C^{(1)}_{1,2,3});$
  $A_3^{(2)} \leftarrow A_3^{(1)} - p_1.(C^{(1)}_{1,\overline{2},3} + C^{(1)}_{1,2,3});$
  /* Allocations on l_2 */
  $r_2 \leftarrow \lceil d_1 \times S/\phi_2 \rceil;$
  $a_2 \leftarrow \min(r_2, A_2^{(2)});$
  $d_2 \leftarrow \min(d_1, a_2 \times \phi_2/S);$
  $p_2 \leftarrow a_2/A_2^{(2)};$
  $A_3^{(3)} \leftarrow A_3^{(2)} - p_2.[C^{(1)}_{\overline{1},2,3} + (1 - p_1).C^{(1)}_{1,2,3}];$
  /* Allocations on l_3*/
  $r_3 \leftarrow \lceil d_2 \times S/\phi_3 \rceil;$
  $a_3 \leftarrow \min(r_3, A_3^{(3)});$
  $d_3 \leftarrow \min(d_2, a_3 \times \phi_3/S);$

Clique 1: The clique 1 is the easiest to process as it does not depend on any previous allocations. As described in Section 4, exactly $a_1 = \min(r_1, A_1^{(1)})$ slots are reserved for communication on link l_1. Then, the slots remaining available for communication on l_1 are not considered anymore and the calculation is passed on to clique 2.

Clique 2: To process any clique i, we calculate a_i by first estimating $A_i^{(i)}$, the average number of slots remaining available for reservation on link l_i just before n_i does its allocations. Given the 3-link available slot set decomposition of clique i, we get:

$$A_i^{(i)} = C^{(i)}_{i,\overline{i+1},\overline{i+2}} + C^{(i)}_{i,i+1,\overline{i+2}} + C^{(i)}_{i,\overline{i+1},i+2} + C^{(i)}_{i,i+1,i+2} \quad (5)$$

Indeed, all the resulting sets of the decomposition are disjointed and form a partition of the global slot set $\{1, ..., S.B\}$. Then, to correctly measure the impact caused on clique 2 sets by reservations done on l_1, we just extend the 3-link available slot set decomposition related to clique 2 to the 4-link decomposition including l_1. This way, we note that:

$$C^{(1)}_{2,\overline{3},\overline{4}} = \underbrace{C^{(1)}_{1,2,\overline{3},\overline{4}}}_{\substack{\text{impacted by} \\ \text{allocations on } l_1}} + \underbrace{C^{(1)}_{\overline{1},2,\overline{3},\overline{4}}}_{\text{not impacted}} \quad (6)$$

From this equation, we infer that an average proportion $p_1 = a_1/A_1^{(1)}$ of slots in $E^{(1)}_{1,2,\overline{3},\overline{4}}$ is likely to become unavailable for reservation on link l_2 after node n_1 performs its allocations for communication on link l_1. Using this principle and considering that there is no interference between l_1 and l_4, the clique 2 sets can be updated as follows:

$$\begin{array}{ll}
C^{(2)}_{2,\overline{3},\overline{4}} = C^{(1)}_{2,\overline{3},\overline{4}} - p_1.C^{(1)}_{1,2,\overline{3},\overline{4}} & C^{(2)}_{2,\overline{3},4} = C^{(1)}_{2,\overline{3},4} - p_1.C^{(1)}_{1,2,\overline{3},4} \\
C^{(2)}_{\overline{2},3,\overline{4}} = C^{(1)}_{\overline{2},3,\overline{4}} - p_1.C^{(1)}_{1,\overline{2},3,\overline{4}} & C^{(2)}_{\overline{2},3,4} = C^{(1)}_{\overline{2},3,4} - p_1.C^{(1)}_{1,\overline{2},3,4} \\
C^{(2)}_{2,3,\overline{4}} = C^{(1)}_{2,3,\overline{4}} - p_1.C^{(1)}_{1,2,3,\overline{4}} & C^{(2)}_{2,3,4} = C^{(1)}_{2,3,4} - p_1.C^{(1)}_{1,2,3,4} \\
\multicolumn{2}{l}{C^{(2)}_{\overline{2},\overline{3},4} = C^{(1)}_{\overline{2},\overline{3},4} + p_1.\left(C^{(1)}_{1,2,\overline{3},4} + C^{(1)}_{1,\overline{2},3,4} + C^{(1)}_{1,2,3,4}\right)} \\
\multicolumn{2}{l}{C^{(2)}_{\overline{2},\overline{3},\overline{4}} = C^{(1)}_{\overline{2},\overline{3},\overline{4}} + p_1.\left(C^{(1)}_{1,2,\overline{3},\overline{4}} + C^{(1)}_{1,\overline{2},3,\overline{4}} + C^{(1)}_{1,2,3,\overline{4}}\right)}
\end{array} \tag{7}$$

As depicted in the previous equations, some sets receive new slots. This phenomenon results from slot allocations on l_1 having a different impact on slots initially available for reservation on links l_2, l_3 and l_4. Indeed, due to the 2-hop nature of the interference, a proportion of slots that were initially available in common for l_1, l_2, l_3 and l_4 have become unavailable to l_2 and l_3 and thus become exclusively available to l_4. At this point, it is possible to correctly calculate the average values of $A^{(2)}_2$, r_2, a_2, d_2 and $p_2 = a_2/A^{(2)}_2$.

Clique 3: Exactly the same interference phenomenon occurs when processing the third clique. However, as this clique suffers from interference created by allocations on both previous links l_1 and l_2, the same approach needs to be followed by extending the available slot set decomposition including these two links. It is even more complex than that since (1) l_1 interferes only with l_3, not l_4 and (2) l_3 suffers from interferences created by allocations on both l_1 and l_2 as:

$$\begin{aligned} C^{(1)}_{3,4,5} = {} & \underbrace{C^{(1)}_{1,2,3,4,5}}_{\text{impacted by allocations on } l_1 \text{ and then } l_2} + \underbrace{C^{(1)}_{1,\overline{2},3,4,5}}_{\text{impacted by allocations on } l_1 \text{ but not } l_2} \\ & + \underbrace{C^{(1)}_{\overline{1},2,3,4,5}}_{\text{impacted by allocations on } l_2 \text{ but not } l_1} + \underbrace{C^{(1)}_{\overline{1},\overline{2},3,4,5}}_{\text{not impacted}} \end{aligned} \tag{8}$$

that leads to:

$$\begin{aligned} C^{(3)}_{3,4,5} = C^{(1)}_{3,4,5} & - \left[p_1 + p_2(1-p_1)\right].C^{(1)}_{1,2,3,4,5} \\ & - p_1.C^{(1)}_{1,\overline{2},3,4,5} - p_2.C^{(1)}_{\overline{1},2,3,4,5} \end{aligned} \tag{9}$$

Measuring the impact of allocations on l_1 and l_2 is equivalent to transferring slots from one set to another. Indeed, from the previous equations, we can conclude that on average $p_1.\left[C^{(1)}_{1,2,3,4,5} + C^{(1)}_{1,\overline{2},3,4,5}\right]$ slots and $\left[p_2(1-p_1).C^{(1)}_{1,2,3,4,5} + p_2.C^{(1)}_{\overline{1},2,3,4,5}\right]$ slots in the set $E^{(1)}_{3,4,5}$ are respectively reserved on links l_1 and l_2. Due to the 2-hop nature of the interference, when updating the sets resulting from the 3-link decomposition related to clique 3, on average $p_1.\left[C^{(1)}_{1,2,3,4,5} + C^{(1)}_{1,\overline{2},3,4,5}\right]$ slots from the set $E^{(1)}_{3,4,5}$ are transferred to the set $E^{(1)}_{\overline{3},4,5}$ and $\left[p_2(1-p_1).C^{(1)}_{1,2,3,4,5} + p_2.C^{(1)}_{\overline{1},2,3,4,5}\right]$ to the set $E^{(1)}_{\overline{3},\overline{4},5}$. Every set resulting from the 3-link available slot set decomposition related to clique 3 is then similarly updated. More generally speaking, when processing the i^{th} clique, the influence of allocations on the two previous links can be correctly considered by updating the sets resulting from its 3-link available slot set decomposition as follows:

$$\mathbf{C}^{(i)}_i = \mathbf{C}^{(1)}_i - \mathbf{p}_i \times \mathbf{I}_i + \mathbf{u}_i + \mathbf{v}_i \tag{10}$$

where

$$\begin{aligned} \mathbf{C}^{(j)}_i &= \begin{pmatrix} C^{(j)}_{\overline{i},\overline{i+1},\overline{i+2}} & C^{(j)}_{i,\overline{i+1},\overline{i+2}} & \cdots & C^{(j)}_{i,i+1,i+2} \end{pmatrix} \\ \mathbf{p}_i &= \begin{pmatrix} p_{i-2} & p_{i-1} & \left[p_{i-2} + p_{i-1}.(1-p_{i-2})\right] \end{pmatrix} \end{aligned}$$

and

$$\mathbf{I}_i = \begin{pmatrix} C^{(i-2)}_{i-2,\overline{i-1},\overline{i},\overline{i+1},\overline{i+2}} & \cdots & C^{(i-2)}_{i-2,\overline{i-1},i,i+1,i+2} \\ C^{(i-2)}_{\overline{i-2},i-1,\overline{i},\overline{i+1},\overline{i+2}} & \cdots & C^{(i-2)}_{\overline{i-2},i-1,i,i+1,i+2} \\ C^{(i-2)}_{i-2,i-1,\overline{i},\overline{i+1},\overline{i+2}} & \cdots & C^{(i-2)}_{i-2,i-1,i,i+1,i+2} \end{pmatrix} \tag{11}$$

The vector $\mathbf{u}_i$ serves to compensate a set that does not suffer from all of the interferences. The vector $\mathbf{v}_i$ is then used to update the sets receiving slots becoming unavailable for reservation on certain links. The values of vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ depend on some variables used in $\mathbf{p}_i$ and $\mathbf{I}_i$ but are not presented in this paper due to space limitations.
Once the clique sets are updated, the average values $A^{(i)}_i$, r_i, a_i, d_i and $p_i = a_i/A^{(i)}_i$ can be correctly evaluated and the calculation process can be passed on to the following clique.

Clique 4 and beyond: When processing the third clique, the entries of matrix $\mathbf{I}_i$ were strictly referring to sets that had not varied from the beginning of the estimation process. However, that is not the case when processing the fourth clique. Indeed, the corresponding sets are likely to have been impacted by allocations on previous links. Such a set, for instance $E^{(2)}_{2,\overline{3},4,5,6}$, has suffered from allocations on l_1 and thus needs also to be updated. A straight solution would consist in forming the sets resulting from the 6-link available slot set decomposition and identify the way every set is impacted. This method is correct but leads to the previously mentioned domino effect. Fortunately, the random nature of the slot allocation can simplify the analysis and bound the number of variables to deal with for each clique process. In the following, we illustrate this point when processing any clique $i \geq 4$. We now show how to characterize a set used in the clique i set update equation, say $E^{(i-2)}_{i-2,\overline{i-1},i,i+1,i+2}$, as a function of its initial state. We start by doing the 2-link available slot set decomposition related to l_{i-2} and l_{i-1}. This decomposition leads to four disjointed sets: $E^{(j)}_{\overline{i-2},\overline{i-1}}$, $E^{(j)}_{\overline{i-2},i-1}$, $E^{(j)}_{i-2,\overline{i-1}}$ and $E^{(j)}_{i-2,i-1}$. Let us work on the third one, that is $E^{(j)}_{i-2,\overline{i-1}}$. This set can also be divided in eight disjointed subsets resulting from the 3-link available slot set decomposition of clique i. This time, the slot space equals $E^{(j)}_{i-2,\overline{i-1}}$ rather than $\{1, 2, ..., S.B\}$, leading to subsets of the following form $E^{(j)}_{i-2,\overline{i-1},i,i+1,i+2}$, taken as an example. A property of the set $E^{(j)}_{i-2,\overline{i-1}}$ is that along the estimation process, it can only transfer slots to the set $E^{(j)}_{\overline{i-2},\overline{i-1}}$ and cannot receive slots from another. Therefore, the number of slots that initially belonged to the set $E^{(j)}_{i-2,\overline{i-1}}$ and had become unavailable just before node n_{i-2} did its reservations for communication on link l_{i-2} equals $C^{(1)}_{i-2,\overline{i-1}} - C^{(i-2)}_{i-2,\overline{i-1}}$. These slots had become unavailable due to allocations on l_{i-4} and l_{i-3}. Because of the random nature of the slot reservation process, these slots were taken uniformly at random among the subsets partitioning $E^{(j)}_{i-2,\overline{i-1}}$. We can thus represent the number of slots that had become unavailable in the set $E^{(j)}_{i-2,\overline{i-1},i,i+1,i+2}$ with the discrete random variable $X_{i-2,\overline{i-1},i,i+1,i+2}$ taking its values in the set $\{0, ..., C^{(1)}_{i-2,\overline{i-1}} - C^{(i-2)}_{i-2,\overline{i-1}}\}$ and following an hypergeometric distribution with parameters $(C^{(1)}_{i-2,\overline{i-1}}, C^{(1)}_{i-2,\overline{i-1},i,i+1,i+2}, C^{(1)}_{i-2,\overline{i-1}} - C^{(i-2)}_{i-2,\overline{i-1}})$. From this identification we can deduce that, for $C^{(1)}_{i-2,\overline{i-1}}$ strictly positive, the average value of this random variable equals $[(C^{(1)}_{i-2,\overline{i-1}} - C^{(i-2)}_{i-2,\overline{i-1}})/C^{(1)}_{i-2,\overline{i-1}}] \times C^{(1)}_{i-2,\overline{i-1},i,i+1,i+2}$. More generally, just before node n_{i-2} did its allocations, an average proportion $C^{(i-2)}_{i-2,\overline{i-1}}/C^{(1)}_{i-2,\overline{i-1}}$ of the initially available slots remained available in every subset partitioning $E^{(j)}_{i-2,\overline{i-1}}$. Hereafter, the quantity $C^{(i-2)}_{i-2,\overline{i-1},i,i+1,i+2}$ can be correctly evaluated as follows:

$$C^{(i-2)}_{i-2,\overline{i-1},i,i+1,i+2} = C^{(1)}_{i-2,\overline{i-1},i,i+1,i+2} \times \alpha_{i-2,\overline{i-1}} \quad (12)$$

where the reduction factor of the set $E^{(j)}_{i-2,\overline{i-1}}$ equals

$$\alpha_{i-2,\overline{i-1}} = \begin{cases} 0 \quad , \text{if } C^{(1)}_{i-2,\overline{i-1}} = 0 \\ \frac{C^{(i-2)}_{i-2,\overline{i-1}}}{C^{(1)}_{i-2,\overline{i-1}}} = \frac{C^{(i-2)}_{i-2,\overline{i-1},\overline{i}} + C^{(i-2)}_{i-2,\overline{i-1},i}}{C^{(1)}_{i-2,\overline{i-1},\overline{i}} + C^{(1)}_{i-2,\overline{i-1},i}} \quad , \text{else} \end{cases} \quad (13)$$

and is related to clique $(i-2)$ as it can be computed at the beginning of the process of this clique. The same analysis can be carried out for the two other sets of interest $E^{(j)}_{\overline{i-2},i-1}$ and $E^{(j)}_{i-2,i-1}$. However, it differs a little for $E^{(j)}_{\overline{i-2},i-1}$ when $C^{(1)}_{\overline{i-2},i-1} = 0$ as this set can receive slots from $E^{(j)}_{i-2,i-1}$ due to allocations on previous links. For this case, to correctly update the resulting subsets, it suffices to compute the proportion of slots transferred from $E^{(j)}_{i-2,i-1}$ to $E^{(j)}_{\overline{i-2},i-1}$.

Then, no additional techniques are required to process the remaining cliques and the calculation can be completed by simply applying the approach given in algorithm 1 when processing the last clique of the path. The main advantage of this approach is that there is no domino effect and the resulting calculation complexity is $\mathcal{O}(N_H)$.

6. PERFORMANCE EVALUATION

We have evaluated the proposed calculation algorithm in simulations using MATLAB R2012b.

Simulation parameters: Each link is assigned one orthogonal channel among four sensed ones. The medium is accessed through a TDMA MAC with 40 slots per frame. As the spectrum assignment process is beyond the scope of this paper, we simply use the following probabilistic model to select the assigned channel for each communication link: $\text{P}[channel1] = 0.80$, $\text{P}[channel2] = 0.10$, $\text{P}[channel3] = 0.05$ and $\text{P}[channel4] = 0.05$. Such probabilities are voluntarily chosen to approach the worst-case scenario when link transmissions are occurring on the same channel along the path, hence maximizing intra-flow interference. Then, as the operating link rates depend on the local environment and may fluctuate, we sample every link rate ϕ_i according to a normal distribution with mean μ_ϕ and standard deviation σ_ϕ where μ_ϕ is the mean link transmission rate on the corresponding assigned channel and σ_ϕ is taken

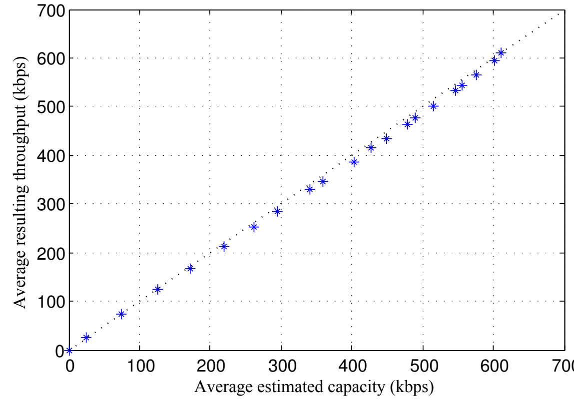


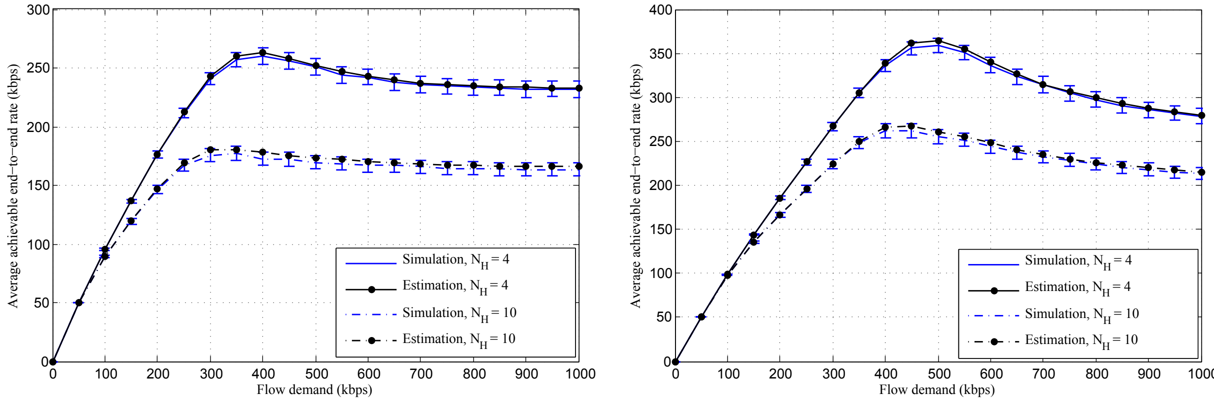
Figure 3: This figure represents, for different values of p_a and $\sigma_\phi = \mu_\phi \times 0.10$, the average achieved throughput on 4-hop paths when trying to admit a flow with demand equal to the capacity estimate. It shows nearly 100% admission for every flow.

proportional to μ_ϕ. For the four channels considered, we choose: $\mu_\phi(channel1) = 2\text{Mbps}$, $\mu_\phi(channel2) = 1.5\text{Mbps}$, $\mu_\phi(channel3) = 800\text{kbps}$ and $\mu_\phi(channel4) = 250\text{kbps}$. During our analysis, we collected results for paths with available slots for reservation distributed randomly and uniformly among the links. We refer to the proportion of slots available on the path as p_a. We average the results on a sufficient number of runs to get satisfactorily 95% confidence intervals.

Accuracy: For evaluating the accuracy of our method, we compare for any existing slot allocations and input demand, the estimated average achievable end-to-end throughput with the one obtained by simulation. As depicted in Figure 2, using average quantities in our calculation have not degraded the estimation accuracy. According to an extensive numerical analysis we performed for 10-hop and 20-hop paths, this statement still holds for longer paths. Then, as our goal is to provide admission control, we expect any flow with demand equal to the capacity estimate to be admitted end-to-end. To verify that this is the case we perform the following experiment. We consider a given multi-hop path and compute its residual end-to-end capacity using our algorithm. Then we run a simulation using this particular path and try to perform an end-to-end assignment with demand equal to the output of our computation. As shown in Figure 3, nearly 100% admission was achieved. That means that almost all the time, the algorithm calculated an accurate and achievable end-to-end residual capacity which provides the capability of doing admission control.

7. CONCLUSIONS

In this paper, we have proposed a linear time algorithm for estimating the available end-to-end throughput in TDMA-based multi-hop cognitive radio networks wherein each node is equipped with multiple transceivers. We have addressed the particular case of random slot selection at the MAC layer and provided an admission control scheme for end-to-end flows. Our method is based on the introduced *l-link available slot set decomposition* and the *clique sliding approach*, an approximation scheme that reduces the calculation complexity from exponential to linear while still returning accurate and reliable results.



(a) $p_a = 33\%$ (b) $p_a = 50\%$

Figure 2: The figures represent the average achievable end-to-end throughput as a function of the demand for the cases of 4-hop and 10-hop paths when 33% and 50% of the slots are available. These results have been obtained for $\sigma_\phi = \mu_\phi \times 0.10$. The error bars associated with the average simulation results represent the corresponding 95% confidence interval and demonstrate the accuracy of our estimation algorithm.

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